INTRODUCTION

In the past decades, PID controllers became a fundamental part of many automatic systems. The successful design of PID controller was mostly based on deterministic methods that involve complex mathematics [1, 2]. Recently, different soft-computing methods were used with promising results for solving the task of PID controller design [3]. These stochastic techniques [5-11] use random operations and typically use various kinds of pseudo-random number generators (PRNGs) that depend on the platform the algorithm is implemented. More recently it was shown that discrete chaotic systems could be used as PRNGs for various stochastic methods with great results. Some of these chaos driven stochastic methods were tested on the task of PID controller design in [4]. In [3] it was shown that PSO algorithm could deal with the task of PID controller design with very good results. Following that in [13 - 16] the performance of chaos driven PSO algorithm was tested on this task with great results. In this paper two more promising chaotic systems are investigated and results are compared with previously published results of chaos driven PSO [13 - 16] and other methods [3,4].

PARTICLE SWARM OPTIMIZATION

The PSO algorithm is inspired by the natural swarm behavior of birds and fish. It was introduced by Eberhart and Kennedy in 1995 [5, 9] as an alternative to other ECTs, such as Ant Colony Optimization [6], Genetic Algorithms (GA) [7] or Differential Evolution (DE) [8]. Each particle in the population represents a possible solution of the optimization problem which is defined by its cost function. In each iteration, a new location (combination of cost function parameters) of the particle is calculated based on its previous location and velocity vector (velocity vector contains particle velocity for each dimension of the problem).

One of the disadvantages of the original PSO algorithm was poor local search ability. For this reason, several modifications of the PSO were introduced. The main principles of the PSO algorithm and its modifications are detailed in [5, 10, 11]. Within this research, the chaos driven PSO strategy with linear decreasing inertia weight was utilized [10, 11]. This strategy was first introduced in 1998 [10] in order to improve the local search capability of PSO. The selection of inertia weight strategy of PSO was based on numerous previous experiments [13, 16]. Default values of all PSO parameters were chosen according to the recommendations given in [5, 9, 10, 11].

Inertia weight is designed to influence the velocity of each particle differently over time [10, 11]. In the beginning of the optimization process, the influence of inertia weight factor w is minimal. As the optimization continues, the value of w is decreasing, thus the velocity of each particle is decreasing, since w is always the number less than one and it multiplies the previous velocity of particle in the process of new velocity value calculation. Inertia weight modification PSO strategy has two control parameters $w_{\text{start}}$ and $w_{\text{end}}$. A new w for each iteration is given by (1), where $i$ stands for current iteration number and $n$ stands for the total number of iterations.

\[ w = \frac{w_{\text{start}} - w_{\text{end}}}{n} + w_{\text{end}} \]
A chaos driven PRNG is used in the main PSO formula Eq. 2 that determines new “velocity” and thus the position of each particle in the next iteration.

\[ v(i+1) = w \cdot v(i) + c_1 \cdot \text{Rand} \cdot (p\text{Best} - x(i)) + c_2 \cdot \text{Rand} \cdot (g\text{Best} - x(i)) \]  

(2)

Where:
- \( v(i + 1) \) - New velocity of a particle.
- \( v(i) \) - Current velocity of a particle.
- \( c_1, c_2 \) - Priority factors.
- \( p\text{Best} \) - Best solution found by a particle.
- \( g\text{Best} \) - Best solution found in a population.
- \( x(i) \) - Current position of a particle.
- \( \text{Rand} \) - Random number, interval (0, 1). Chaos PRNG is applied only here.

The new position of a particle is then given by Eq. 3, where \( x(i + 1) \) is the new particle position:

\[ x(i + 1) = x(i) + v(i + 1) \]  

(3)

**CHAOTIC MAPS**

This section contains the description of four discrete chaotic maps used as PRNGs for the PSO algorithm.

**FIGURE 1.** \( x-y \) plots of maps: Lozi (upper left), Dissipative (upper right), Burgers (below left) and Tinkerbell (below right).
Chaotic Lozi Map

The Lozi map is a simple discrete two-dimensional chaotic map. The Lozi map is depicted in Fig. 1. The map equations are given in Eq. 4. Typical parameters given in literature [12] are $a = 1.7$ and $b = 0.5$.

$$
X_{n+1} = 1 - a |X_n| + b Y_n
$$

$$
Y_{n+1} = X_n
$$

Dissipative Standard Map

The Dissipative Standard map is a two-dimensional chaotic map. The parameters used in this work are $b = 0.6$ and $k = 8.8$ as suggested in [12]. The map equations are given in Eq. 5. The $x,y$ plot of Dissipative map is depicted on Fig 1.

$$
X_{n+1} = X_n + Y_n (\mod 2\pi)
$$

$$
Y_{n+1} = b Y_n + k \sin X_n (\mod 2\pi)
$$

Burgers Chaotic Map

The Burgers map (Fig. 1) is a discretization of a pair of coupled differential equations The map equations are given in Eq. 6 with control parameters $a = 0.75$ and $b = 1.75$ as suggested in [12]. The direct output iterations of the map are transferred into the interval $(0, 1)$.

$$
X_{n+1} = a X_n - Y_n^2
$$

$$
Y_{n+1} = b Y_n + X_n Y_n
$$

Tinkerbell Map

The Tinkerbell map is a two-dimensional complex discrete-time dynamical system given by (7) and depicted in Fig. 1 with following control parameters: $a = 0.9$, $b = -0.6$, $c = 2$ and $d = 0.5$ [12].

$$
X_{n+1} = X_n^2 - Y_n^2 + a X_n + b Y_n
$$

$$
Y_{n+1} = 2 X_n Y_n + c X_n + d Y_n
$$

PROBLEM DESIGN

The PID controller contains three unique parts: proportional, integral and derivative controller [1-4]. A simplified form in Laplace domain is given in (8).

$$
G(s) = K\left(1 + \frac{1}{sT_i} + sT_d\right)
$$

The PID form most suitable for analytical calculations is given in (9).

$$
G(s) = k_p + \frac{k_i}{s} + k_d s
$$

The parameters are related to the standard form through: $k_p = K$, $k_i = K/T_i$ and $k_d = KT_d$. Acquisition of the combination of these three parameters that gives the lowest value of the test criterions was the objective of this research.

The DC motor system used in this research is given by (10).

$$
G(s) = \frac{0.9}{0.00105 s^3 + 0.2104 s^2 + 0.8913 s}
$$

Test criterion measures properties of output transfer function and can indicate quality of regulation [1-4]. Following four different integral criterions were used for the test and comparison purposes: IAE (Integral Absolute Error), ITAE (Integral Time Absolute Error), ISE (Integral Square Error) and MSE (Mean Square Error). These test criterions (given by Eq. 11–14) were minimized within the cost functions for the enhanced PSO algorithm.

Integral of Time multiplied by Absolute Error (ITAE)

$$
I_{ITAE} = \int_0^T |e(t)| dt
$$

Integral of Absolute Magnitude of the Error (IAE)

$$
I_{IAE} = \int_0^T |e(t)| dt
$$

Integral of the Square of the Error (ISE)

$$
I_{ISE} = \int_0^T e^2(t) dt
$$

Mean of the Square of the Error (MSE)

$$
I_{MSE} = \frac{1}{n} \sum_{t=1}^{n} e(t)^2
$$

RESULTS

In this section results obtained in the experiments with PSO algorithm driven by selected set of four chaotic maps are compared with previously published works [3, 4, 13 - 16]. Best results obtained for each
method either proposed here or founded in the literature are given in Table 1. The best overall results are highlighted by bold numbers.

The selected examples of PID controller designs with the respect to the best criterion values and step response characteristics are given in Table 2.

The example of obtained system step responses is depicted on Fig. 2.

<table>
<thead>
<tr>
<th>TABLE 1. PID controller designs comparison.</th>
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<tbody>
<tr>
<td>Criterion/Method</td>
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<tr>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>Z-N (step response)</td>
</tr>
<tr>
<td>Kappa-Tau</td>
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<tr>
<td>Continuous cycling</td>
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<tr>
<td>EP</td>
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<tr>
<td>GA</td>
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<tr>
<td>PSO</td>
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<tr>
<td>PSO Lozi</td>
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<tr>
<td>PSO Disi</td>
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<tr>
<td>PSO Burger</td>
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<td>PSO Tinker</td>
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<table>
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<th>TABLE 2. PID controller design – an example.</th>
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<tbody>
<tr>
<td>Criterion/Constant value</td>
</tr>
<tr>
<td>---------------------------------------------</td>
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<tr>
<td>$K_p$</td>
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<tr>
<td>$K_i$</td>
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<td>$K_d$</td>
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DISCUSSION

Data presented in tables in previous section seem to support the claims that using chaotic systems as PRNGs for the PSO algorithm could improve the performance of such algorithm on the task of PID controller design. Furthermore it seems that these algorithm are capable of very successful PID controller designs as the step response (Fig. 2) confirms (the system stabilization is very fast).

In comparison with deterministic methods, the results obtained by the chaos driven PSO algorithm are significantly better. In comparison with other stochastic methods, the results are better or comparable.

Furthermore Fig. 2 supports the claim that using different chaotic systems could lead to different results not only in numerical value of the criterion but in different behavior of the controlled process. Depicted example step response (Fig. 2) for the ITAE criterion...
slightly differ in terms of the overshoot value, rise time and other values important for control systems design. Nevertheless the main aim of this research was not to investigate deeply within the numerical results of the step response characteristics. But to show that embedding of the chaotic natural behavior into the evolutionary/swarm based algorithms may help to obtain better results.

These findings may show important for future using of stochastic methods, driven by chaotic sequences for the PID controller design. Overall, it is possible to claim, that through usage of Burgers map or Tinkerbell map as the chaotic PRNG for evolutionary or swarm based algorithm it is possible to obtain better results than with any compared meta-heuristic.

CONCLUSION

In this paper four different chaotic systems were presented and investigated over their capability of enhancing the performance of PSO algorithm in the task of PID controller design. The experimental systems of DC motor was described and used for the PID controller design. Promising results were presented, discussed and compared with other methods of PID controller design. It was claimed that it seems that using chaotic sequences in the PSO algorithm could improve its performance on the task of PID controller design. More detail experiments are needed to prove or disprove these claims and explain the effect of the chaotic systems on the optimization and controller design.

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